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DEVELOPMENT OF SIMULATION TECHNIQUES FOR THE M739 AND M577 FUZES

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This report consists of two parts.

Part 1 outlines test plans for an experimental investigation to verify existing computer simulations of the M577 SSD and the M739 safing and arming mechanism. Special emphasis is placed on the determination of the influence of the variation of the escapement center distance on the number-of-turns-to-arm of these devices. Part 2 gives complete forward as well as inverse kinematic models of the two meshes of a two-pass clock gear train of an artillery S&A (cont)

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mechanism. In the forward model, gears no. 1 and 2 represent the drivers while in the inverse model, pinion no. 2 and the escapewheel pinion act as the input components.

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INTRODUCTION

This project consisted of two separate tasks.

Task 1 involved the development of test plans for an experimental investigation to verify existing computer simulations of the M577 safe separation device (SSD) and the M739 safing and arming (S&A) mechanism.

Task 2 dealt with the formulation of the forward and inverse kinematic models of the type of two-pass clock gear train used in artillery S&A mechanisms.

TEST PLANS

Test Plan 1--Experimental Investigation Pertaining to Influence of Variation of Escapement Center Distance on Number-of-Turns-to-Arm of the M577 SSD

This test plan consists of the following main phases:

1. Measurements of certain crucial dimensions of M577 SSD housings and mechanism train components associated with specific units. Each manufacturer should supply, measure, and record a sufficient number of units.

2. Selective assembly of components by all manufacturers with the aim of obtaining full SSD assemblies which only vary with respect to the escapement center distances (C.D.'s). (If needed, certain subgroups of SSD assemblies are to be formed within which only the escapement C.D.'s vary. This accounts for rotor property variations.)

3. Conductance of spin tests of SSDs under the following conditions:

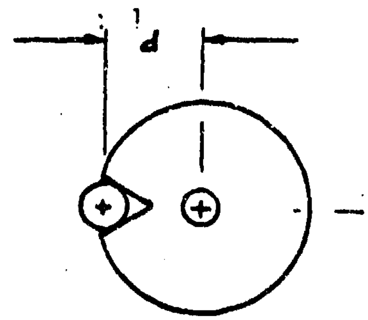
   a. Spin axis coinciding with SSD axis

   b. Spin axis not coinciding with SSD axis (magnitude of eccentricity and orientation of SSD to be recorded)

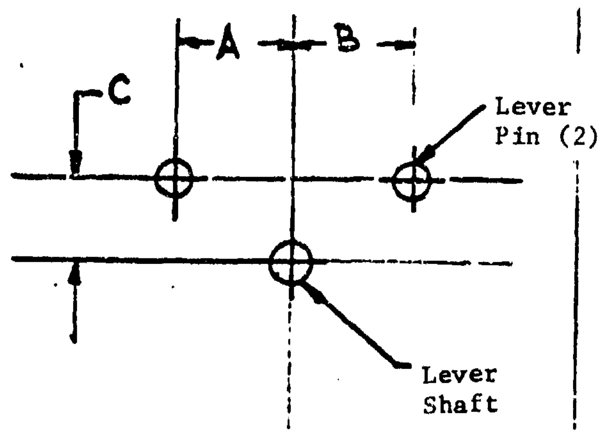
Regarding phase 1, the following actions should be taken:

1. The SSD spacer and plate assembly (dwg 9236552) and the top plate (dwg 9236553) should be put together without rolling in the top plate. Pins of the largest possible diameter should be inserted into the pallet and escape wheel pivot holes and the overwire dimensions should be measured on both sides. (Wire size should be given.)

2. Diameters of both pivots should be measured on the escape wheel assembly (dwg 9236537). The thickness and outside diameter of the escapement gear (star wheel) should be measured and the entire assembly should be weighed. The indicated dimension "d" should be checked at four places, 90 degrees apart, to obtain the tooth angle (wire size should be given).



3. Diameters of both pivots of the lever assembly (dwg 9236540) should be measured. The thickness and outside diameter of the lever should be measured and the entire assembly should be weighed. Dimensions marked as "A", "B", and "C" should be measured and lever pin diameters should be determined.



4. The complete rotor assembly (dwg 9236528) should be weighed. The location of the center of mass (c.g.) with reference to the line connecting the pivot centerline and the centerline of the detonator sleeve hole should be determined.

Test Plan 2--Experimental Investigation Pertaining to Influence of Variation of Escapement Center Distance on Number-of-Turns-to-Arm of the M739 S&A

This test plan consists of the following main phases:

1. Measurements of certain crucial dimensions of M739 S&A housings and mechanism train components associated with specific units. Each manufacturer should supply, measure, and record a sufficient number of units.

2. Selective assembly of components by all manufacturers with the aim of obtaining full S&A assemblies which only vary with respect to the escapement C.D.'s. (If needed, certain subgroups of S&A assemblies are to be formed within which only the escapement C.D.'s vary. This accounts for rotor property variations.)

3. Conductance of spin tests of S&As under the following conditions:

a. Spin axis coinciding with S&A axis

b. Spin axis not coinciding with S&A axis (magnitude of eccentricity and orientation of S&A to be recorded)

Phase 2 consists of the following:

1. For given assemblies of bottom plate (dwg 9258644), gear plate spacer (dwg 9258646), lower plate and shaft assembly (dwg 9258650), and upper plate (dwg 9258632), the following measurements should be determined and recorded:

The overwire C.D. between pallet shaft and the escape wheel pivot holes in the upper plate (the largest wires possible should be used and the wire diameters of the individual holes should be recorded).

The pivot diameter of the pallet shaft

2. For the escape wheel and pinion assembly (dwg 9258655), the following measurements should be taken and recorded:

The outside diameter and thickness of the escape wheel

The weight of the assembly

3. The diameter of the pivot hole in the pallet and the thickness and outside diameter of the pallet (dwg 9258631) should be determined and recorded. Accuracy of the 17 degree, 10 minute angle noted on the drawing should be verified; also the accuracy of the 0.1905-inch dimension to the theoretical corners and the accuracy of the 0.0905-inch dimension. The weight of the entire pallet should be taken and recorded.

4. The weight of the rotor assembly (dwg 9258639) should be taken and recorded. Location of the c.g. should be ascertained with respect to the same axes as given in drawing 9258639.

MATHEMATICAL MODELS OF CLOCK GEAR TRAIN S&A

The following sections provide the basis for a complete mathematical model of an artillery S&A mechanism containing a two-pass clock gear train and a straight-sided verge runaway escapement. They are based on the "Fuze Gear Train Analysis" conducted in 1979 (ref 1).

Complete forward, as well as inverse kinematic models of the two meshes of the clock gear train, have been derived. In the forward model, gears no. 1 and 2 represent the drivers, while in the inverse model, pinion no. 2 and the escapewheel pinion represent the input components.

The inverse model is needed to express all component angular accelerations in terms of the escapewheel angular acceleration.

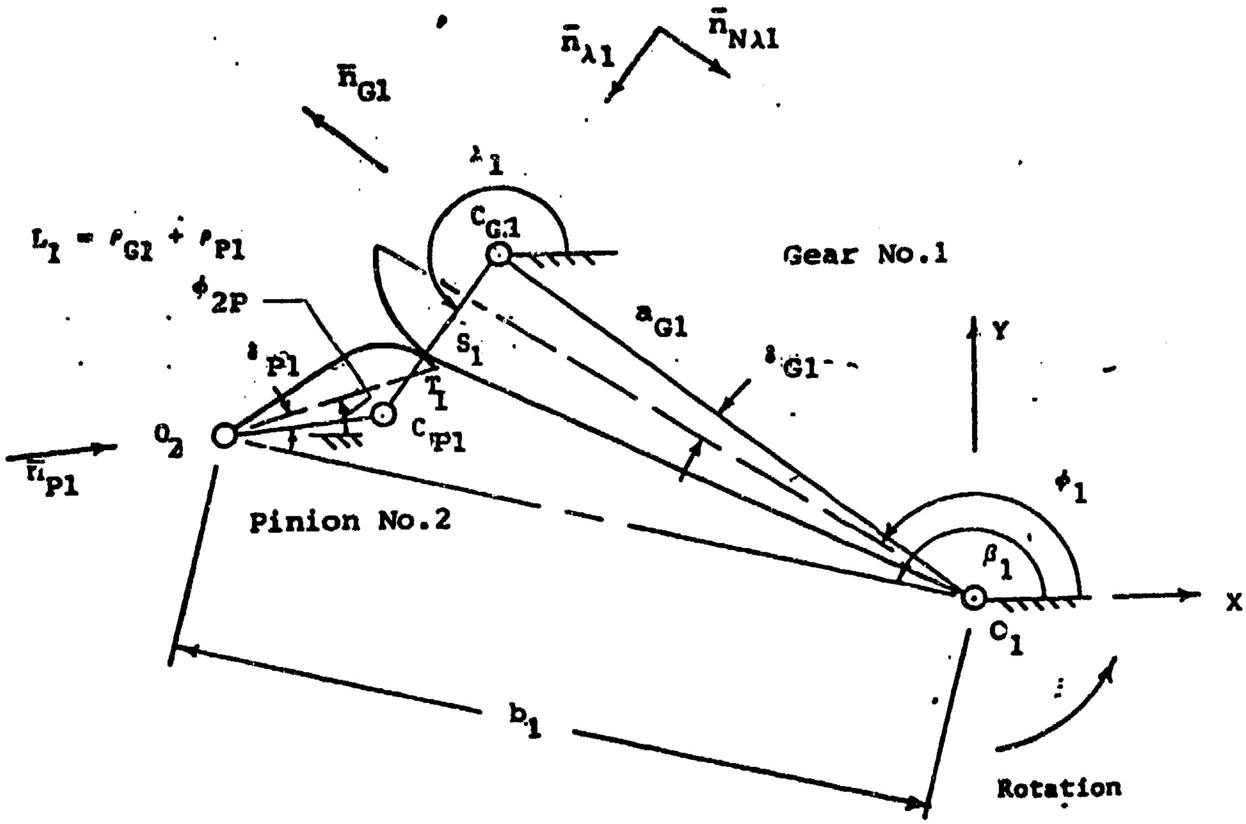


Figure 1. Round-on-round phase of motion for mesh no. 1

FORWARD KINEMATICS OF MESH NO. 1
(Angle ϕ_1 is Input, Angle ϕ_{2P} is Output)[1]

Round-On-Round Phase of Motion

A schematic of mesh no. 1 in round-on-round motion is given in figure 1. The gear no. 1 and the rotor have counterclockwise rotation. The input angle is called ϕ_1, while the output angle is called ϕ_{2P}. The following derivation, where ϕ_{2P} is found as a function of ϕ_1, runs parallel to reference 1, appendix G-2a.

Unit Vectors

The unit vector in the direction O_1C_{G1} of the gear no. 1 is given by

$$\bar{n}_{G1} = \cos\left(\phi_1 - \delta_{G1}\right)\bar{I} + \sin\left(\phi_1 - \delta_{G1}\right)\bar{J} \tag{1}$$

The unit vector in the direction of $C_{G1}C_{P1}$ is given by

$$\bar{n}_{\lambda 1} = \cos\lambda_1\bar{I} + \sin\lambda_1\bar{J} \tag{2}$$

The unit vector normal to $\bar{n}_{\lambda 1}$ in the right hand sense becomes

$$\bar{n}_{N\lambda 1} = -\sin\lambda_1\bar{I} + \cos\lambda_1\bar{J} \tag{3}$$

The pinion unit vector $\bar{n}_{P1}$ in the direction of O_2C_{P2}, is represented by:

$$\bar{n}_{P1} = \cos\left(\phi_{2P} - \delta_{P1}\right)\bar{I} + \sin\left(\phi_{2P} - \delta_{P1}\right)\bar{J} \tag{4}$$

Finally, the unit vector in the direction from pivot O_1 to pivot O_2 is given by

$$\bar{n}_{\beta 1} = \cos\beta_1\bar{I} + \sin\beta_1\bar{J} \tag{5}$$

---

[1] The output angle of mesh 1 is called ϕ_{2P} (P = pinion), while the input angle of mesh 2 will be ϕ_{2G} (G = gear). Only the increments of these angles are equal.

Output Angle ϕ_{2P} and "Coupler" Angle λ_1

The loop equation of the equivalent four-bar linkage $O_1C_{G1}C_{P1}O_2$ is given by

$$a_{G1}\,\bar{n}_{G1} + L_1\,\bar{n}_{\lambda 1} - a_{P1}\,\bar{n}_{P1} - b_1\,\bar{n}_{\beta 1} = 0 \tag{6}$$

where

$$L_1 = \rho_{G1} + \rho_{P1} \tag{7}$$

After substitution of the various unit vectors, as given earlier, the following component equations are obtained:

$$a_{G1}\cos(\phi_1 - \delta_{G1}) + L_1\cos\lambda_1 - a_{P1}\cos(\phi_{2P} - \delta_{P1}) - b_1\cos\beta_1 = 0 \tag{8}$$

and

$$a_{G1}\sin(\phi_1 - \delta_{G1}) + L_1\sin\lambda_1 - a_{P1}\sin(\phi_{2P} - \delta_{P1}) - b_1\sin\beta_1 = 0 \tag{9}$$

To solve for the output angle ϕ_{2P} in terms of the input angle ϕ_1, substitute the expressions for $\sin\lambda_1$ and $\cos\lambda_1$, as obtained from equations 8 and 9, into

$$\sin^2\lambda_1 + \cos^2\lambda_1 = 1 \tag{10}$$

This leads to

$$A_{1R}\sin\phi_{2P} + B_{1R}\cos\phi_{2P} = C_{1R} \tag{11}$$

where

$$A_{1R} = b_1\sin(\beta_1 + \delta_{P1}) - a_{G1}\sin(\phi_1 - \delta_{G1} + \delta_{P1}) \tag{12}$$

$$B_{1R} = b_1\cos(\beta_1 + \delta_{P1}) - a_{G1}\cos(\phi_1 - \delta_{G1} + \delta_{P1}) \tag{13}$$

$$C_{1R} = \frac{L_1^2 - b_1^2 - a_{G1}^2 - a_{P1}^2 + 2\,a_{G1}\,b_1\cos(\phi_1 - \delta_{G1} - \beta_1)}{2\,a_{P1}} \tag{14}$$

Equation 11 is solved for ϕ_{2P} according to a method used in reference 1.

$$\phi_{2P} = 2\tan^{-1}\frac{A_{1R} \pm \sqrt{A_{1R}^2 + B_{1R}^2 - C_{1R}^2}}{B_{1R} + C_{1R}} \tag{15}$$

The correct sign must be determined by geometric considerations.

The angle λ_1 may now be obtained from equation 8 or 9, i.e.,

$$\lambda_1 = \cos^{-1}\left[\frac{b_1 \cos \beta_1 + a_{p1} \cos(\phi_{2P} - \delta_{p1}) - a_{G1} \cos(\phi_1 - \delta_{G1})}{L_1}\right] \tag{16}$$

or

$$\lambda_1 = \sin^{-1}\left[\frac{b_1 \sin \beta_1 + a_{p1} \sin(\phi_{2P} - \delta_{p1}) - a_{G1} \sin(\phi_1 - \delta_{G1})}{L_1}\right] \tag{17}$$

Output Angular Velocity[2]--$\dot{\phi}_2 = \dot{\phi}_{2P} = \dot{\phi}_{2G}$

Implicit differentiation of equation 11 with respect to time furnishes the angular velocity $\dot{\phi}_2$

$$\dot{\phi}_2 = \dot{\phi}_1\left[\frac{A_{1RD} \sin \phi_{2P} - B_{1RD} \cos \phi_{2P} - C_{1RD}}{A_{1R} \cos \phi_{2P} - B_{1R} \sin \phi_{2P}}\right] \tag{18}$$

where

$$A_{1RD} = a_{G1} \cos(\phi_1 - \delta_{G1} + \delta_{P1})$$

$$B_{1RD} = a_{G1} \sin(\phi_1 - \delta_{G1} + \delta_{P1})$$

$$C_{1RD} = \frac{a_{G1}\, b_1 \sin(\phi_1 - \delta_{G1} - \beta_1)}{a_{P1}}$$

or

$$\dot{\phi}_2 = \dot{\phi}_1\left[\frac{L_{1R}}{M_{1R}}\right] \tag{19}$$

---

[2] Regarding the derivatives of the gear and pinion no. 2, there is no difference whether the gear or the pinion is involved. The difference is only needed for the angles, since the angles ϕ_{2P} and ϕ_{2G} are expressed with respect to different center lines.

where

$$L_{1R} = A_{1RD} \sin \phi_{2P} - B_{1RD} \cos \phi_{2P} - C_{1RD} \tag{20}$$

$$M_{1R} = A_{1R} \cos \phi_{2P} - B_{1R} \sin \phi_{2P} \tag{21}$$

Relative Velocity at the Contact Point

The relative velocity $\bar{V}_{S1/T1_R}$ of point S_1 on gear no. 1 with respect to T_1 on pinion no. 2 has the direction of unit vector $\bar{n}_{N\lambda 1}$. The resulting expression may be adapted from reference 1, equation G-63.

$$\bar{V}_{S1/T1_R} = \{\dot{\phi}_1 \left[a_{G1} \cos \left(\phi_1 - \delta_{G1} - \lambda_1\right) + \rho_{G1}\right]$$

$$- \dot{\phi}_2 \left[a_{P1} \cos \left(\phi_{2P} - \delta_{P1} - \lambda_1\right) - \rho_{P1}\right]\}\bar{n}_{N\lambda 1} \tag{22}$$

Round-On-Flat Phase of Motion

Figure 2 shows a schematic view of mesh no. 1 in the round-on-flat phase of the motion. (Only the contacting sides of the gear teeth are indicated.) The following work runs parallel to that described in reference 1, appendix G2b:

Unit Vectors

The unit vector in the direction O_2T_1, along the flank of pinion no. 2, is given by

$$\bar{n}_{F1} = \cos \left(\phi_{2P} + \alpha_{P1}\right)\bar{I} + \sin \left(\phi_{2P} + \alpha_{P1}\right)\bar{J} \tag{23}$$

The unit vector $\bar{n}_{NF1}$ in the direction S_1C_{G1} is normal to $\bar{n}_{F1}$ in the right hand sense.

$$\bar{n}_{NF1} = - \sin \left(\phi_{2P} + \alpha_{P1}\right)\bar{I} + \cos \left(\phi_{2P} + \alpha_{P1}\right)\bar{J} \tag{24}$$

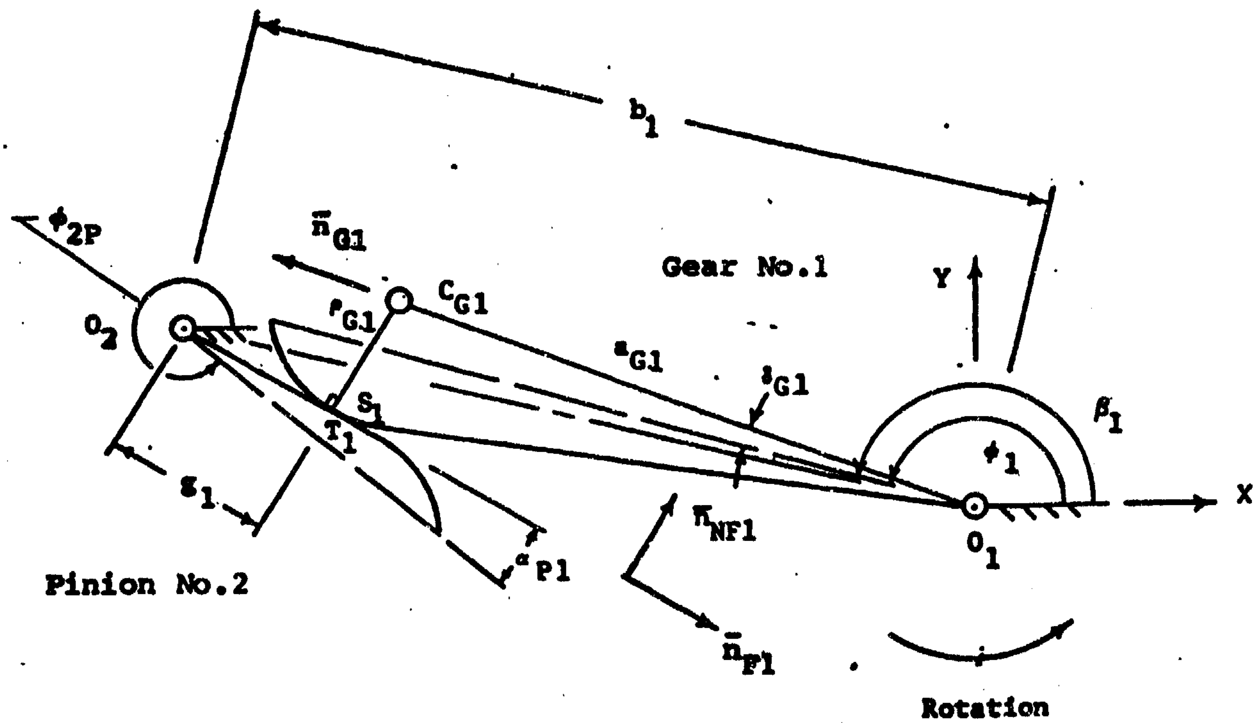


Figure 2. Round-on-flat phase of motion of mesh no. 1

Output Angle ϕ_{2P} and Distance g_1

The vector equation for the mechanism loop O_1-C_{G1}-S_1-T_1-O_2 has the form

$$a_{G1}\ \bar{n}_{G1} - \rho_{G1}\ \bar{n}_{NF1} - g_1\ \bar{n}_{F1} - b_1\ \bar{n}_{\beta 1} = 0 \tag{25}$$

Appropriate substitutions for the unit vectors furnish the following component equations

$$a_{G1} \cos\left(\phi_1 - \delta_{G1}\right) + \rho_{G1} \sin\left(\phi_{2P} + \alpha_{P1}\right) - b_1 \cos \beta_1$$
$$- g_1 \cos\left(\phi_{2P} + \alpha_{P1}\right) = 0 \tag{26}$$

and

$$a_{G1} \sin\left(\phi_1 - \delta_{G1}\right) - \rho_{G1} \cos\left(\phi_{2P} + \alpha_{P1}\right) - b_1 \sin \beta_1$$
$$- g_1 \sin\left(\phi_{2P} + \alpha_{P1}\right) = 0 \tag{27}$$

From equation 27:

$$g_1 = \frac{a_{G1} \sin\left(\phi_1 - \delta_{G1}\right) - \rho_{G1} \cos\left(\phi_{2P} + \alpha_{P1}\right) - b_1 \sin \beta_1}{\sin\left(\phi_{2P} + \alpha_{P1}\right)} \tag{28a}$$

Substitute equation 28 into equation 26

$$\left[a_{G1} \cos\left(\phi_1 - \delta_{G1}\right) + \rho_{G1} \sin\left(\phi_{2P} + \alpha_{P1}\right) - b_1 \cos \beta_1\right] \sin\left(\phi_{2P} + \alpha_{P1}\right)$$
$$- \left[a_{G1} \sin\left(\phi_1 - \delta_{G1}\right) - \rho_{G1} \cos\left(\phi_{2P} + \alpha_{P1}\right) - b_1 \sin \beta_1\right] \cos\left(\phi_{2P} + \alpha_{P1}\right) = 0 \tag{28b}$$

Simplification leads to

$$A_{1F} \sin \phi_{2P} + B_{1F} \cos \phi_{2P} = C_{1F} \tag{29}$$

where

$$A_{1F} = a_{G1} \cos\left(\phi_1 - \delta_{G1} - \alpha_{P1}\right) - b_1 \cos\left(\beta_1 - \alpha_{P1}\right)$$

$$B_{1F} = - a_{G1} \sin (\phi_1 - \delta_{G1} - \alpha_{P1}) + b_1 \sin (\beta_1 - \alpha_{P1})$$

$$C_{1F} = - \rho_{G1}$$

Finally

$$\phi_{2P} = 2 \tan^{-1} \frac{A_{1F} \pm \sqrt{A_{1F}^2 + B_{1F}^2 - C_{1F}^2}}{B_{1F} + C_{1F}} \tag{30}$$

The appropriate sign is found from geometric considerations.

Determination of Angular Velocity $\dot{\phi}_2 = \dot{\phi}_{2P} = \dot{\phi}_{2G}$ for Round-On-Flat Phase of Motion

Differentiation of equation 29 with respect to time gives for $\dot{\phi}_2$

$$\dot{\phi}_2 = \dot{\phi}_1 \left[\frac{A_{1FD} \sin \phi_{2P} + B_{1FD} \cos \phi_{2P}}{A_{1F} \cos \phi_{2P} - B_{1F} \sin \phi_{2P}} \right] \tag{31}$$

where

$$A_{1FD} = a_{G1} \sin (\phi_1 - \delta_{G1} - \alpha_{P1})$$

$$B_{1FD} = a_{G1} \cos (\phi_1 - \delta_{G1} - \alpha_{P1})$$

Relative Velocity $\bar{V}_{S1/T1_F}$ at Contact Point During Round-On-Flat Phase of Motion

As shown in reference 1, the relative velocity $\bar{V}_{S1/T1_F}$ consists only of that component of $\bar{V}_{S1/01}$ which is directed along the flank of pinion no. 2. Thus,

$$\bar{V}_{S1/T1_F} = [\bar{V}_{S1/01} \cdot \bar{n}_{F1}]\, \bar{n}_{F1}$$

$$= \{[\dot{\phi}_1 \bar{k} \times (a_{G1} \bar{n}_{G1} - \rho_{G1} \bar{n}_{NF1})] \cdot \bar{n}_{F1}\} \bar{n}_{F1} \tag{32}$$

Substitution of appropriate unit vectors gives

$$\bar{V}_{S1/T1_F} = \dot{\phi}_1 \{a_{G1} \sin (\phi_{2P} + \alpha_{P1} - \phi_1 + \delta_{G1}) + \rho_{G1}\} \bar{n}_{F1} \tag{33}$$

Determination of Transition Angles

The transition angles ϕ_{1T} and ϕ_{2PT}, which correspond to the transition from the round-on-round to the round-on-flat phase of the motion, are determined by letting $g_1 = f_{P1}$ in the component equations 26 and 27. From this, the following is found:

$$\cos (\phi_{1T} - \delta_{G1}) = \frac{1}{a_{G1}} [-\rho_{G1} \sin (\phi_{2PT} + \alpha_{P1}) + b_1 \cos \beta_1 + f_{P1} \cos (\phi_{2PT} + \alpha_{P1})] \tag{34}$$

and

$$\sin (\phi_{1T} - \delta_{G1}) = \frac{1}{a_{G1}} [\rho_{G1} \cos (\phi_{2PT} + \alpha_{P1}) + b_1 \sin \beta_1 + f_{P1} \sin (\phi_{2PT} + \alpha_{P1})] \tag{35}$$

The angle ϕ_{2PT} is now found by substituting the above two expressions into

$$\sin^2 (\phi_{1T} - \delta_{G1}) + \cos^2 (\phi_{1T} - \delta_{G1}) = 1 \tag{36}$$

This results in

$$A_{1T} \sin \phi_{2PT} + B_{1T} \cos \phi_{2PT} = C_{1T} \tag{37}$$

where

$$A_{1T} = - \rho_{G1} \cos (\beta_1 - \alpha_{P1}) + f_{P1} \sin (\beta_1 - \alpha_{P1})$$

$$B_{1T} = \rho_{G1} \sin (\beta_1 - \alpha_{P1}) + f_{P1} \cos (\beta_1 - \alpha_{P1})$$

$$C_{1T} = \frac{a_{G1}^2 - \rho_{G1}^2 - b_1^2 - f_{P1}^2}{2b_1}$$

Finally, equation 36 is solved for ϕ_{2PT} in the usual manner of reference 1.

$$\phi_{2PT} = 2 \tan^{-1} \frac{A_{1T} \pm \sqrt{A_{1T}^2 + B_{1T}^2 - C_{1T}^2}}{B_{1T} + C_{1T}} \tag{38}$$

Here also, the sign must be decided from geometric considerations.

The associated angle ϕ_{1T} may be found with the help of equation 34 or equation 35; i.e.,

$$\phi_{1T} = \cos^{-1}\left[\frac{-\rho_{G1} \sin(\phi_{2PT} + \alpha_{P1}) + b_1 \cos\beta_1 + f_{P1}\cos(\phi_{2PT} + \alpha_{P1})}{a_{G1}}\right] + \delta_{G1} \tag{39}$$

$$\phi_{1T} = \sin^{-1}\left[\frac{\rho_{G1} \cos(\phi_{2PT} + \alpha_{P1}) + b_1 \sin\beta_1 + f_{P1}\sin(\phi_{2PT} + \alpha_{P1})}{a_{G1}}\right] + \delta_{G1} \tag{40}$$

Sensing Equations for the Determination of Contact on Subsequent Tooth Mesh

The following contact sensing equation for mesh no. 1 is based on the method originally shown in reference 1, appendix E. Again, it is assumed that the contact of the new mesh will be made in the round-on-round mode. Before contact, the distance between the centers of curvature C_{G1} and C_{P1} is given by

$$C_{G1}\, C_{P1} = L_{x1}\bar{i} + L_{y1}\bar{j} \tag{41}$$

If $\Delta\phi_1$ and $\Delta\phi_{2P}$ represent the tooth spacing angles of gear no. 1 and pinion no. 2 respectively, the associated loop equation becomes (see fig. E-3 of reference 1 together with fig. 1 of this report)

$$\begin{aligned} & a_{G1}\left[\cos(\phi_1 - \Delta\phi_1 - \delta_{G1})\bar{i} + \sin(\phi_1 - \Delta\phi_1 - \delta_{G1})\bar{j}\right] \\ & + L_x\bar{i} + L_y\bar{j} - a_{P1}\left[\cos(\phi_{2P} + \Delta\phi_{2P} - \delta_{P1})\bar{i}\right. \\ & \left. + \sin(\phi_{2P} + \Delta\phi_{2P} - \delta_{P1})\bar{j}\right] - b_1(\cos\beta_1\bar{i} + \sin\beta_1\bar{j}) = 0 \end{aligned} \tag{42}$$

Note that for mesh no. 1, the angular increment $\Delta\phi_1$ is negative, while $\Delta\phi_{2P}$ is positive. Further, as in reference 1, the angle ϕ_{2P} must be determined for the round-on-flat phase of motion, since the initial contact of a subsequent set of meshing teeth is preceded by this phase of motion. This means that equation 30 is applicable.

The instantaneous magnitudes of L_{x1} and L_{y1} are obtained from the components of equation 42; i.e.,

$$L_{x1} = b_1 \cos \beta_1 + a_{P1} \cos \left(\phi_{2P} + \Delta\phi_{2P} - \delta_{P1}\right) - a_{G1} \cos \left(\phi_1 - \Delta\phi_1 - \delta_{G1}\right) \tag{43}$$

and

$$L_{y1} = b_1 \sin \beta_1 + a_{P1} \sin \left(\phi_{2P} + \Delta\phi_{2P} - \delta_{P1}\right) - a_{G1} \sin \left(\phi_1 - \Delta\phi_1 - \delta_{G1}\right) \tag{44}$$

Contact will occur as soon as

$$\sqrt{L_{x1}^2 + L_{x2}^2} < \rho_{G1} + \rho_{P1} \tag{45}$$

FORWARD KINEMATICS OF MESH NO. 2
(Angle ϕ_{2G} is Input, Angle ϕ is Output)

Round-On-Round Phase of Motion

A diagram of mesh no. 2 in the round-on-round phase of contact is shown in figure 3. Gear and pinion no. 2 have clockwise rotation, while pinion no. 3, together with the escape wheel, has counterclockwise rotation. The input angle associated with gear no. 2 is called ϕ_{2G}, while the output angle of the escape wheel is defined as ϕ. The following derivation runs parallel to appendix G-1a of reference 1:

Unit Vectors

The unit vector in direction O_2C_{G2} of gear no. 2 is given by

$$\bar{n}_{G2} = \cos \left(\phi_{2G} + \delta_{G2}\right)\bar{I} + \sin \left(\phi_{2G} + \delta_{G2}\right)\bar{J} \tag{46}$$

The unit vector in direction $C_{G2}C_{P2}$ is given by

$$\bar{n}_{\lambda 2} = \cos \lambda_2 \bar{I} + \sin \lambda_2 \bar{J} \tag{47}$$

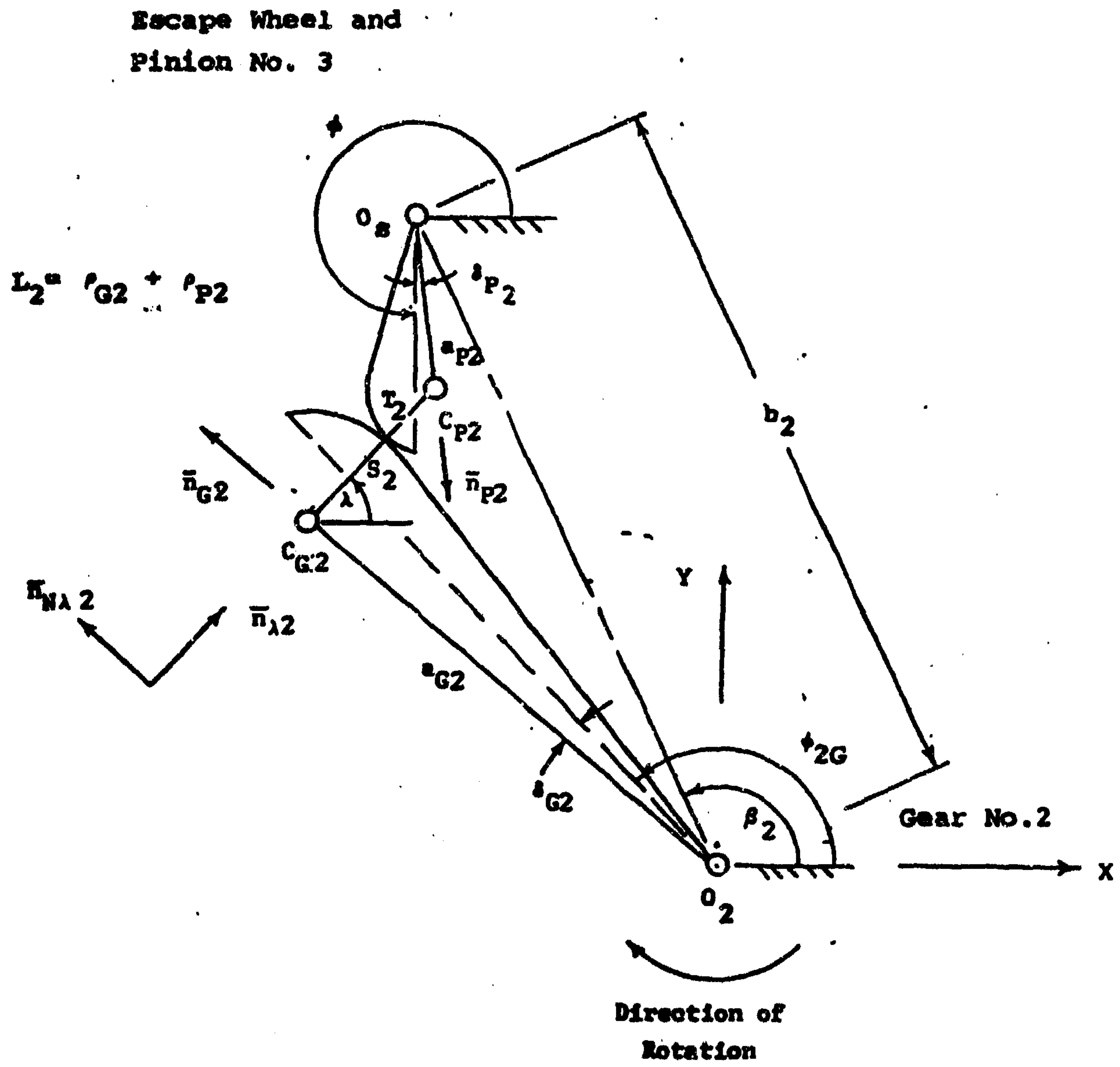


Figure 3. Round-on-round phase of motion for mesh no. 2

The unit vector normal to $\bar{n}_{\lambda 2}$ (in the right hand sense) becomes

$$\bar{n}_{N\lambda 2} = -\sin\lambda_2\bar{i} + \cos\lambda_2\bar{j} \qquad (48)$$

The unit vector in direction O_sC_{P2} is given by

$$\bar{n}_{P2} = \cos(\phi + \delta_{P2})\bar{I} + \sin(\phi + \delta_{P2})\bar{J} \qquad (49)$$

Finally, the unit vector along the centerline from O_2 to O_s is given by

$$\bar{n}_{\beta 2} = \cos\beta_2\bar{i} + \sin\beta_2\bar{j} \qquad (50)$$

Determination of Output Angle ϕ and "Coupler" Angle λ_2

The loop equation of the equivalent four-bar linkage $O_2C_{G2}C_{P2}O_s$ is given by

$$a_{G2}\,\bar{n}_{G2} + L_2\,\bar{n}_{\lambda 2} - a_{p2}\,\bar{n}_{P2} - b_2\,\bar{n}_{\beta 2} = 0 \qquad (51)$$

where

$$L_2 = \rho_{G2} + \rho_{P2} \qquad (52)$$

After substitution of the various unit vectors into equation 51, the following component equations are obtained:

$$a_{G2}\cos(\phi_{2G} + \delta_{G2}) + L_2\cos\lambda_2 - b_2\cos\beta_2 - a_{p2}\cos(\phi + \delta_{P2}) = 0 \qquad (53)$$

$$a_{G2}\sin(\phi_{2G} + \delta_{G2}) + L_2\sin\lambda_2 - b_2\sin\beta_2 - a_{p2}\sin(\phi + \delta_{P2}) = 0 \qquad (54)$$

To eliminate λ_2 let:

$$\sin^2\lambda_2 + \cos^2\lambda_2 = 1 \qquad (55)$$

The resulting expression furnishes (ref 1)

$$A_{2R} \sin \phi + B_{2R} \cos \phi = C_{2R} \tag{56}$$

where

$$A_{2R} = a_{G2} \sin \left(\phi_{2G} + \delta_{G2} - \delta_{P2}\right) - b_2 \sin \left(\beta_2 - \delta_{2P}\right)$$

$$B_{2R} = a_{G2} \cos \left(\phi_{2G} + \delta_{G2} - \delta_{P2}\right) - b_2 \cos \left(\beta_2 - \delta_{P2}\right)$$

$$C_{2R} = \frac{a_{P2}^2 + a_{G2}^2 + b_2^2 - L_2^2 - 2a_{G2}\, b_2 \cos \left(\phi_{2G} + \delta_{G2} - \beta_2\right)}{2a_{P2}}$$

Equation 56 is now solved for ϕ according to a method used in reference 1:

$$\phi = 2 \tan^{-1} \frac{A_{2R} \pm \sqrt{A_{2R}^2 + B_{2R}^2 - C_{2R}^2}}{B_{2R} + C_{2R}} \tag{57}$$

The correct sign in the above must be determined by geometric considerations.

The angle λ_2 may now be obtained with the help of equations 53 and 54

$$\lambda_2 = \cos^{-1}\left[\frac{b_2 \cos \beta_2 + a_{P2} \cos \left(\phi + \delta_{P2}\right) - a_{G2} \cos \left(\phi_{2G} + \delta_{G2}\right)}{L_2}\right] \tag{58}$$

or

$$\lambda_2 = \sin^{-1}\left[\frac{b_2 \sin \beta_2 + a_{P2} \sin \left(\phi + \delta_{P2}\right) - a_{G2} \sin \left(\phi_{2G} + \delta_{G2}\right)}{L_2}\right] \tag{59}$$

Determination of Angular Velocity $\dot{\phi}$ of Escape Wheel for Round-On-Round Motion

Differentiation of equation 56 with respect to time furnishes

$$\dot{\phi} = \dot{\phi}_{2G} \left[\frac{B_{2RD} \cos \phi - A_{2RD} \sin \phi + C_{2RD}}{A_{2R} \cos \phi - B_{2R} \sin \phi}\right] \tag{60}$$

where

$$A_{2RD} = a_{G2} \cos \left(\phi_{2G} + \delta_{G2} - \delta_{P2}\right)$$

$$B_{2RD} = a_{G2} \sin (\phi_{2G} + \delta_{G2} - \delta_{P2})$$

$$C_{2RD} = \frac{a_{G2}\, b_2 \sin (\phi_{2G} + \delta_{G2} - \beta_2)}{a_{P2}}$$

Note that in equation 60

$$\dot{\phi}_{2G} = \dot{\phi}_{2P} = \dot{\phi}_2 \tag{61}$$

Relative Velocity at the Contact Point

The relative velocity $V_{S2/T2_R}$ of the contact point S_2 on gear no. 2 with respect to point T_2 on pinion no. 3 may be adapted with equation G21 of reference 1

$$\bar{V}_{S2/T2_R} = \{\dot{\phi}_{2G} [a_{G2} \cos (\phi_{2G} + \delta_{G2} - \lambda_2) + \rho_{G2}]$$
$$- \dot{\phi} [a_{P2} \cos (\phi + \delta_{P2} - \lambda_2) - \rho_{P2}]\} \bar{n}_{N\lambda 1} \tag{62}$$

Note that in above

$$\dot{\phi}_{2G} = \dot{\phi}_2$$

Round-On-Flat Phase of Motion

A schematic view of mesh no. 2 in the round-on-flat phase of the motion is shown in figure 4.

Unit Vectors

The unit vector in the direction $O_s T_2$ is given by

$$\bar{n}_{F2} = \cos (\phi - \alpha_{P2}) \bar{I} + \sin (\phi - \alpha_{P2}) \bar{J} \tag{63}$$

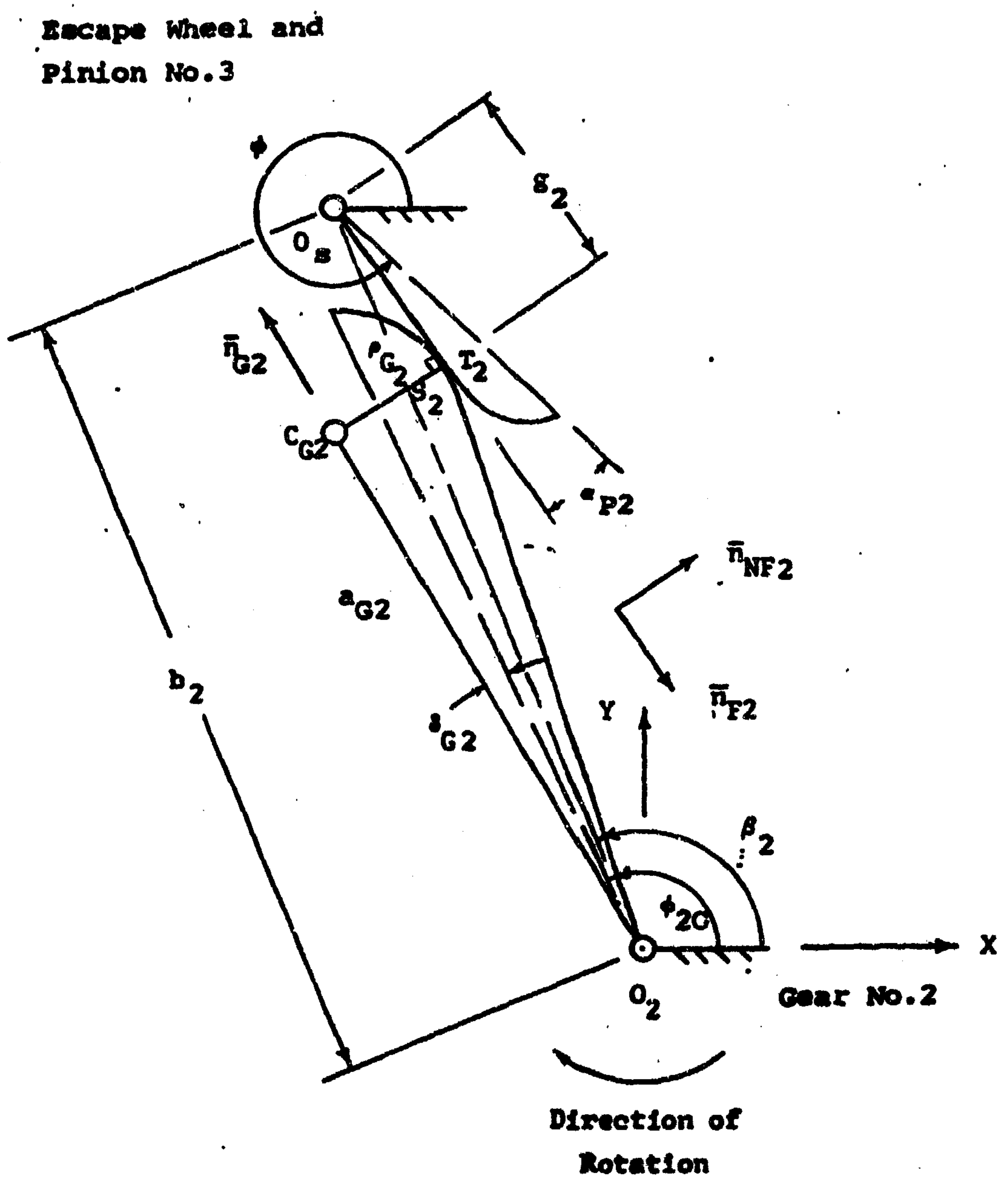


Figure 4. Round-on-flat phase of motion for mesh no. 2

The unit vector $\bar{n}_{NF2}$, in the direction $C_{G2}S_2$ is always normal to $\bar{n}_{F2}$. Thus,

$$\bar{n}_{NF2} = -\sin(\phi - \alpha_{P2})\bar{I} + \cos(\phi - \alpha_{P2})\bar{J} \tag{64}$$

Determination of Angle ϕ and Distance g_2 in Terms of Angle ϕ_{2G}

The vector equation for the mechanism loop $O_2C_{G2}S_2T_2O_s$ has the form:

$$a_{G2}\,\bar{n}_{G2} + \rho_{G2}\,\bar{n}_{NF2} - g_2\,\bar{n}_{F2} - b_2\,\bar{n}_{\beta 2} = 0 \tag{65}$$

Substitution of equations 46, 50, 63, and 64 into the above leads to the following component equations:

$$a_{G2}\cos(\rho_{2G} + \delta_{G2}) - \rho_{G2}\sin(\phi - \alpha_{P2}) - b_2\cos\beta_2 - g_2\cos(\phi - \alpha_{P2}) = 0 \tag{66}$$

and

$$a_{G2}\sin(\phi_{2G} + \delta_{G2}) + \rho_{G2}\cos(\phi - \alpha_{P2}) - b_2\sin\beta_2 - g_2\sin(\phi - \alpha_{P2}) = 0 \tag{67}$$

The following is obtained from equation 67:

$$g_2 = \frac{a_{G2}\sin(\phi_{2G} + \delta_{G2}) + \rho_{G2}\cos(\phi - \alpha_{P2}) - b_2\sin\beta_2}{\sin(\phi - \alpha_{P2})} \tag{68}$$

Substitution of this expression into equation 66 results in

$$A_{2F}\sin\phi + B_{2F}\cos\phi = C_{2F} \tag{69}$$

where

$$A_{2F} = a_{G2}\cos(\phi_{2G} + \delta_{G2} + \alpha_{P2}) - b_2\cos(\beta_2 + \alpha_{P2})$$

$$B_{2F} = -a_{G2}\sin(\phi_{2G} + \delta_{G2} + \alpha_{P2}) + b_1\sin(\beta_2 + \alpha_{P2})$$

$$C_{2F} = \rho_{G2}$$

Equation 69 is now solved for ϕ as earlier in reference 1

$$\phi = 2 \tan^{-1} \frac{A_{2F} \pm \sqrt{A_{2F}^2 + B_{2F}^2 - C_{2F}^2}}{B_{2F} + C_{2F}} \tag{70}$$

Determination of Angular Velocity $\dot{\phi}$ During Round-On-Flat Motion

Implicit differentiation of equation 69 with respect to time gives for $\dot{\phi}$

$$\dot{\phi} = \dot{\phi}_{2G} \left[\frac{A_{2FD} \sin \phi + B_{2FD} \cos \phi}{A_{2F} \cos \phi - B_{2F} \sin \phi} \right] \tag{71}$$

where

$$\dot{\phi}_{2G} = \dot{\phi}_2$$

and

$$A_{2FD} = a_{G1} \sin (\phi_{2G} + \delta_{G2} + \alpha_{P2})$$

$$B_{2FD} = a_{G2} \cos (\phi_{2G} + \delta_{G2} + \alpha_{P2})$$

Relative Velocity $\bar{V}_{S2/T2_F}$ at Contact Point During Round-On-Flat Motion Phase

Again, as shown in reference 1, the relative velocity $\bar{V}_{S2/T2_F}$ consists only of that component of $\bar{V}_{S2/O2}$ which is directed along the flank of pinion no. 3. Thus

$$\bar{V}_{S2/T2_F} = [\bar{V}_{S2/O2} \cdot \bar{n}_{F2}]\, \bar{n}_{F2}$$

$$= \{[\dot{\phi}_2 \bar{k} \times (a_{G2}\, \bar{n}_{G2} + \rho_{G2}\, \bar{n}_{NF2})] \cdot \bar{n}_{F2}\}\, \bar{n}_{F2} \tag{72}$$

Substitution of the appropriate unit vectors furnishes

$$\bar{V}_{S2/T2_F} = \dot{\phi}_2 \, [a_{G2} \sin (\phi - \alpha_{P1} - \phi_{2G} - \delta_{G2}) - \rho_{G2}]\, \bar{n}_{F2} \tag{73}$$

Determination of Transition Angles

The transition angles ϕ_{2GT} and ϕ_T are reached when the round-on-round phase of motion is followed by the round-on-flat one. They are obtained by letting $g_2 = f_{p2}$ in the component equations 66 and 67. This furnishes

$$a_{G2} \cos(\phi_{2GT} + \delta_{G2}) - \rho_{G2} \sin(\phi_T - \alpha_{P2}) - b_2 \cos \beta_2 - f_{P2} \cos(\phi_T - \alpha_{P2}) = 0 \tag{74}$$

and

$$a_{G2} \sin(\phi_{2GT} + \delta_{G2}) + \rho_{G2} \cos(\phi_T - \alpha_{P2}) - b_2 \sin \beta_2 - f_{P2} \sin(\phi_T - \alpha_{P2}) = 0 \tag{75}$$

from the above, the following is obtained:

$$\cos(\phi_{2GT} + \delta_{G2}) = \frac{1}{a_{G2}} [\rho_{G2} \sin(\phi_T - \alpha_{P2}) + b_2 \cos \beta_2 + f_{P2} \cos(\phi_T - \alpha_{P2})] \tag{76}$$

and

$$\sin(\phi_{2GT} + \delta_{G2}) = \frac{1}{a_{G2}} [- \rho_{G2} \cos(\phi_T - \alpha_{P2}) + b_2 \cos \beta_2 + f_{P2} \sin(\phi_T - \alpha_{P2})] \tag{77}$$

The transition angle ϕ_T is now obtained by first using

$$\sin^2(\phi_{2GT} + \delta_{G2}) + \cos^2(\phi_{2GT} + \delta_{G2}) = 1 \tag{78}$$

This results in

$$A_{2T} \sin \phi + B_{2T} \cos \phi = C_{2T} \tag{79}$$

where

$$A_{2T} = \rho_{G2} \cos(\beta_2 + \alpha_{P2}) + f_{P2} \sin(\beta_2 + \alpha_{P2})$$

$$B_{2T} = - \rho_{G2} \sin(\beta_2 + \alpha_{P2}) + f_{P2} \cos(\beta_2 + \alpha_{P2})$$

$$C_{2T} = \frac{a_{G2}^2 - \rho_{G2}^2 - b_2^2 - f_{P2}^2}{2b_2}$$

Equation 79 is solved for ϕ in the usual manner. Thus,

$$\phi = 2\tan^{-1}\frac{A_{2T} \pm \sqrt{A_{2T}^2 + B_{2T}^2 - C_{2T}^2}}{B_{2T} + C_{2T}} \tag{80}$$

The appropriate sign must be found from geometric considerations.

The associated angle ϕ_{2G} is determined with the help of equations 76 and 77; i.e.,

$$\phi_{2GT} = \cos^{-1}\left[\frac{\rho_{G2}\sin(\phi_T - \alpha_{P2}) + f_{P2}\cos(\phi_T - \alpha_{P2}) + b_2\cos\beta_2}{a_{G2}}\right] - \delta_{G2} \tag{81}$$

or

$$\phi_{2GT} = \sin^{-1}\left[\frac{-\rho_{G2}\cos(\phi - \alpha_{P2}) + f_{P2}\sin(\phi - \alpha_{P2}) + b_2\sin\beta_2}{a_{G2}}\right] - \delta_{G2} \tag{82}$$

Sensing Equations for the Determination of Contact on Subsequent Tooth Mesh

The following contact sensing expression proceeds from the same assumption as that for mesh no. 1 in the section, "Determination of Angular Velocity $\dot{\phi}_2 = \dot{\phi}_{2P} = \dot{\phi}_{2G}$ for Round-On-Flat Phase of Motion." The configuration is that of figure 3, where gear no. 2 rotates in a CW direction, and initial contact of the new mesh is in the round-on-round mode. Before contact, the distance between the centers of curvature C_{G2} and C_{p2} is given by

$$C_{G2}\,C_{P2} = L_{x2}\bar{i} + L_{y2}\bar{j} \tag{83}$$

If $\Delta\phi_{2G}$ and $\Delta\phi$ represent the tooth spacing angles of gear no. 2 and pinion no. 3, respectively, the associated loop equation becomes (ref 1)

$$a_{G2}\left[\cos(\phi_{2G} + \Delta\phi_{2G} + \delta_{G2})\bar{i} + \sin(\phi_{2G} + \Delta\phi_{2G} + \delta_{G2})\bar{j}\right]$$

$$+ L_{x2}\bar{i} + L_{y2}\bar{j} = b_2\left[\cos\beta_2\bar{i} + \sin\beta_2\bar{j}\right]$$

$$+ a_{P2}\left[\cos(\phi - \Delta\phi + \delta_{P2})\bar{i} + \sin(\phi - \Delta\phi + \delta_{P2})\bar{j}\right]$$

where the angle ϕ of the escape wheel and pinion no. 3 must be determined with the help of equation 70 since the mesh which precedes the new one is in the round-on-flat mode.

The magnitudes of L_{x2} and L_{y2} are determined with the help of the component expressions of equation 84

$$L_{x2} = b_2 \cos \beta_2 + a_{P2} \cos (\phi - \Delta\phi + \delta_{P2}) - a_{G2} \cos (\phi_{2G} + \Delta\phi_{2G} + \delta_{G2}) \tag{85}$$

and

$$L_{y2} = b_2 \sin \beta_2 + a_{P2} \sin (\phi - \Delta\phi + \delta_{P2}) - a_{G2} \sin (\phi_{2G} + \Delta\phi_{2G} + \delta_{G2}) \tag{86}$$

Contact will occur as soon as

$$\sqrt{L_{x2}^2 + L_{y2}^2} < \rho_{G2} + \rho_{P2} \tag{87}$$

INVERSE KINEMATICS OF MESH NO. 1
(Angle ϕ_{2P} is Input and Angle ϕ_1 is Output)

Round-On-Round Phase of Motion

Determination of Angle ϕ_1 in Terms of Angle ϕ_{2P}

The component loop equations 8 and 9 are again used to determine equation 10, i.e.,

$$\sin^2 \lambda + \cos^2 \lambda = 1 \tag{88}$$

Appropriate substitution and expansion furnishes

$$L_1^2 = a_{P1}^2 + b_1^2 + a_{G1}^2 + 2a_{P1}\, b_1 \cos \{\beta_1 + \delta_{P1} - \phi_{2P})$$

$$+ \sin \phi_1 \{- 2a_{P1}\, a_{G1} \sin (\phi_{2P} - \delta_{P1}) \cos \delta_{G1}\}$$

$$- 2a_{P1}\, a_{G1} \cos\left(\phi_{2P} - \delta_{P1}\right) \sin \delta_{G1}$$

$$- 2b_1\, a_{G1} \sin \beta_1 \cos \delta_{G1} - 2b_1\, a_{G1} \cos \beta_1 \sin \delta_{G1} \}$$

$$+ \cos \phi_1 \{+ 2a_{P1}\, a_{G1} \sin\left(\phi_{2P} - \delta_{P1}\right) \sin \delta_{G1}$$

$$- 2a_{P1}\, a_{G1} \cos\left(\phi_{2P} - \delta_{p1}\right) \cos \delta_{G1}$$

$$+ 2b_1\, a_{G1} \sin \beta_1 \sin \delta_{G1} - 2b_1\, a_{G1} \cos \beta_1 \cos \delta_{G1} \} \tag{89}$$

Further simplification gives

$$L_1^2 - a_{P1}^2 - a_{G1}^2 - b_1^2 - 2a_{P1}\, b_1 \cos\left(\beta_1 + \delta_{P1} - \phi_{2P}\right)$$

$$= \sin \phi_1 \left[- 2a_{P1}\, a_{G1} \sin\left(\phi_{2P} - \delta_{P1} + \delta_{G1}\right) - 2b_1\, a_{G1} \sin\left(\beta_1 + \delta_{G1}\right)\right]$$

$$+ \cos \phi_1 \left[- 2a_{P1}\, a_{G1} \cos\left(\phi_{2P} - \delta_{P1} + \delta_{G1}\right) - 2b_1\, a_{G1} \cos\left(\beta_1 + \delta_{G1}\right)\right] \tag{90}$$

The above leads to

$$D_{1R} \sin \phi_1 + E_{1R} \cos \phi_1 = F_{1R}$$

where

$$D_{1R} = - 2a_{G1} \left[a_{P1} \sin\left(\phi_{2P} + \delta_{G1} - \delta_{P1}\right) + b_1 \sin\left(\beta_1 + \delta_{G1}\right)\right]$$

$$E_{1R} = - 2a_{G1} \left[a_{P1} \cos\left(\phi_{2P} + \delta_{G1} - \delta_{P1}\right) + b_1 \cos\left(\beta_1 + \delta_{G1}\right)\right]$$

$$F_{1R} = L_1^2 - a_{G1}^2 - a_{P1}^2 - b_1^2 - 2a_{P1}\, b_1 \cos\left(\phi_{2p} - \beta_1 - \delta_{P1}\right)$$

Equation 91 is now solved in the manner of reference 1; i.e.,

$$\phi_1 = 2 \tan^{-1} \frac{D_{1R} \pm \sqrt{D_{1R}^2 + E_{1R}^2 - F_{1R}^2}}{E_{1R} + F_{1R}} \tag{92}$$

Determination of Angular Velocity $\dot{\phi}_{1R}$ in Terms of Angular Velocity $\dot{\phi}_{2P}$ and Angles ϕ_{2P} and ϕ_1

Implicit differentiation of equation 92 gives

$$D_{1R}\ \dot{\phi}_1 \cos \phi_1 + D_{1RD} \sin \phi_1\ \dot{\phi}_{2P}$$

$$- E_{1R}\ \dot{\phi}_1 \sin \phi_1 + E_{1RD} \cos \phi_1\ \dot{\phi}_{2P} = F_{1RD}\ \dot{\phi}_{2P} \tag{93}$$

where

$$D_{1RD} = -\ 2a_{G1}\ a_{P1} \cos \left(\phi_{2P} + \delta_{G1} - \delta_{P1}\right)$$

$$E_{1RD} = 2a_{G1}\ a_{P1} \sin \left(\phi_{2P} + \delta_{G1} - \delta_{P1}\right)$$

$$F_{1RD} = 2a_{P1}\ b_1 \sin \left(\phi_{2P} - \beta_1 - \delta_{P1}\right)$$

Then,

$$\dot{\phi}_1 \left(D_{1R} \cos \phi_1 - E_{1R} \sin \phi_1\right) = \dot{\phi}_{2P} \left(F_{1RD} - D_{1RD} \sin \phi_1 - E_{1RD} \cos \phi_1\right) \tag{94}$$

Now, after introduction of the additional subscript R, the angular velocity $\dot{\phi}_{1R}$ for the round-on-round phase is given by

$$\dot{\phi}_{1R} = \dot{\phi}_{2P}\ \mathrm{DER1R} \tag{95}$$

where

$$\mathrm{DER1R} = \frac{F_{1RD} - D_{1RD} \sin \phi_1 - E_{1RD} \cos \phi_1}{D_{1R} \cos \phi_1 - E_{1R} \sin \phi_1}$$

Determination of Angular Acceleration $\ddot{\phi}_{1R}$ in Terms of Angular Acceleration $\ddot{\phi}_{2P}$, Angular Velocity $\dot{\phi}_{2P}$, and Angles ϕ_{2P} and ϕ_1

Further differentiation of equation 94 gives

$$\ddot{\phi}_1 \left[D_{1R} \cos \phi_1 - E_{1R} \sin \phi_1\right] + \dot{\phi}_1 \left[\dot{\phi}_{2P}\ D_{1RD} \cos \phi_1\right.$$

$$- \dot{\phi}_{2P} E_{1RD} \sin \phi_1] + \dot{\phi}_1^2 [- D_{1R} \sin \phi_1 - E_{1R} \cos \phi_1]$$

$$= \ddot{\phi}_{2P} [F_{1RD} - D_{1RD} \sin \phi_1 - E_{1RD} \cos \phi_1]$$

$$+ \dot{\phi}_{2P}^2 [F_{1RDD} - D_{1RDD} \sin \phi_1 - E_{1RDD} \cos \phi_1]$$

$$+ \dot{\phi}_{2P} [- D_{1RD} \dot{\phi}_1 \cos \phi_1 + E_{1RD} \dot{\phi}_1 \sin \phi_1]$$

where

$$D_{1RDD} = 2a_{G1} a_{P1} \sin (\phi_{2P} + \delta_{G1} - \delta_{P1})$$

$$E_{1RDD} = 2a_{G1} a_{P1} \cos (\phi_{2P} + \delta_{G1} - \delta_{P1})$$

$$F_{1RDD} = 2a_{P1} b_1 \cos (\phi_{2P} - \beta_1 - \delta_{P1})$$

Substitution of equation 95 into equation 96 and the introduction of the subscript R, where appropriate, leads to the following expression for the angular acceleration $\ddot{\phi}_{1R}$

$$\ddot{\phi}_{1R} = \ddot{\phi}_{2P} X_1 X_2 + \dot{\phi}_{2P}^2 X_1 X_3 \tag{97}$$

where

$$X_1 = \frac{1}{D_{1R} \cos \phi_1 - E_{1R} \sin \phi_1}$$

$$X_2 = F_{1RD} - D_{1RD} \sin \phi_1 - E_{1RD} \cos \phi_1$$

$$X_3 = F_{1RD} - D_{1RDD} \sin \phi_1 - E_{1RDD} \cos \phi_1$$

$$+ \mathrm{DER1R} [2 E_{1RD} \sin \phi_1 - 2 D_{1RD} \cos \phi_1]$$

$$+ \mathrm{DER1R}^2 [D_{1R} \sin \phi_1 + E_{1R} \cos \phi_1]$$

Round-On-Flat Phase of Motion

Determination of Angle ϕ_1 in Terms of Angle ϕ_{2P}

When the terms containing the angle ϕ_1 in equation 28b are trigonometrically expanded, the following is obtained:

$$\begin{aligned}
&\left[a_{G1} \left(\cos \phi_1 \cos \delta_{G1} + \sin \phi_1 \sin \delta_{G1}\right) + \rho_{G1} \sin \left(\phi_{2P} + \alpha_{P1}\right)\right. \\
&\left. - b_1 \cos \beta_1 \right] \sin \left(\phi_{2P} + \alpha_{p1}\right) \\
&- \left[a_{G1} \left(\sin \phi_1 \cos \delta_{G1} - \cos \phi_1 \sin \delta_{G1}\right) - \rho_{G1} \cos \left(\rho_{2P} + \alpha_{P1}\right)\right. \\
&\left. - b_1 \sin \beta_1 \right] \cos \left(\phi_{2P} + \alpha_{P1}\right) = 0
\end{aligned} \tag{98}$$

Further rearrangement furnishes

$$\begin{aligned}
&\left(- a_{G1} \cos \left(\phi_{2P} + \alpha_{P1} + \delta_{G1}\right)\right) \sin \phi_1 + \left(a_{G1} \sin \left(\phi_{2P} + \alpha_{P1} + \delta_{G1}\right)\right) \cos \phi_1 \\
&= - \rho_{G1} + b_1 \sin \left(\phi_{2P} + \alpha_{P1} - \beta_1\right)
\end{aligned} \tag{99}$$

The above may now be written in the following form

$$D_{1F} \sin \phi_1 + E_{1F} \cos \phi_1 = F_{1F} \tag{100}$$

where

$$D_{1F} = - a_{G1} \cos \left(\phi_{2P} + \alpha_{P1} + \delta_{G1}\right)$$

$$E_{1F} = a_{G1} \sin \left(\phi_{2P} + \alpha_{P1} + \delta_{G1}\right)$$

$$F_{1F} = - \rho_{G1} + b_1 \sin \left(\phi_{2P} + \alpha_{P1} - \beta_1\right)$$

Equation 100 is now solved for the angle ϕ_1 in the manner of reference 1 for this type of expression. Thus,

$$\phi_1 = 2 \tan^{-1} \frac{D_{1F} \pm \sqrt{D_{1F}^2 + E_{1F}^2 - F_{1F}^2}}{E_{1F} + F_{1F}} \tag{101}$$

Determination of Angular Velocity $\dot{\phi}_{1F}$ in Terms of Angular Velocity $\dot{\phi}_{2P}$ and Angles ϕ_{2P} and ϕ_1

Implicit differentiation of equation 100 with respect to time furnishes

$$D_{1F}\ \dot{\phi}_1 \cos \phi_1 + D_{1FD}\ \dot{\phi}_{2P} \sin \phi_1$$
$$- E_{1F}\ \dot{\phi}_1 \sin \phi_1 + E_{1FD}\ \dot{\phi}_{2P} \cos \phi_1 = F_{1FD}\ \dot{\phi}_{2P} \qquad (102)$$

where

$$D_{1FD} = a_{G1} \sin \left(\phi_{2P} + \alpha_{P1} + \delta_{G1}\right)$$

$$E_{1FD} = a_{G1} \cos \left(\phi_{2P} + \alpha_{P1} + \delta_{G1}\right)$$

$$F_{1FD} = b_1 \cos \left(\phi_{2P} + \alpha_{P1} - \beta_1\right)$$

Then

$$\dot{\phi}_1\ \left[D_{1F} \cos \phi_1 - E_{1F} \sin \phi_1\right]$$
$$= \dot{\phi}_{2P}\ \left[F_{1FD} - D_{1FD} \sin \phi_1 - E_{1FD} \cos \phi_1\right] \qquad (103)$$

Finally, after the introduction of the additional subscript F, the angular velocity $\dot{\phi}_{1F}$, for the round on flat phase of the motion is given by

$$\dot{\phi}_{1F} = \dot{\phi}_{2P}\ \mathrm{DER1F} \qquad (104)$$

where

$$\mathrm{DER1F} = \frac{F_{1FD} - D_{1FD} \sin \phi_1 - E_{1FD} \cos \phi_1}{D_{1F} \cos \phi_1 - E_{1F} \sin \phi_1}$$

Determination of Angular Acceleration $\ddot{\phi}_{1F}$ in Terms of Angular Acceleration $\ddot{\phi}_{2P}$, Angular Velocity $\dot{\phi}_{2P}$, and Angles ϕ_{2P} and ϕ_1

Implicit differentiation of equation 103 furnishes

$$\ddot{\phi}_1 \left[D_{1F} \cos \phi_1 - E_{1F} \sin \phi_1\right]$$

$$+ \dot{\phi}_1 \left[- D_{1F} \dot{\phi}_1 \sin \phi_1 - E_{1F} \dot{\phi}_1 \cos \phi_1\right]$$

$$+ \dot{\phi}_1 \left[D_{1FD} \cos \phi_1 \dot{\phi}_{2P} - E_{1FD} \sin \phi_1 \dot{\phi}_{2P}\right]$$

$$= \ddot{\phi}_{2P} \left[F_{1FD} - D_{1FD} \sin \phi_1 - E_{1FD} \cos \phi_1\right]$$

$$+ \dot{\phi}_{2P} \left[- D_{1FD} \cos \phi_1 \dot{\phi}_1 + E_{1FD} \dot{\phi}_1 \sin \phi_1\right]$$

$$+ \dot{\phi}^2_{2P} \left[F_{1FDD} - D_{1FDD} \sin \phi_1 - E_{1FDD} \cos \phi_1\right] \qquad (105)$$

where

$$D_{1FDD} = a_{G1} \cos \left(\phi_{2P} + \alpha_{P1} + \delta_{G1}\right)$$

$$E_{1FDD} = - a_{G1} \sin \left(\phi_{2P} + \alpha_{P1} + \delta_{G1}\right)$$

$$F_{1FDD} = - b_1 \sin \left(\phi_{2P} + \alpha_{P1} - \beta_1\right)$$

Substitution of equation 104, and further use of the additional subscript F for round-on-flat motion, leads to the following expression for the angular acceleration ϕ_{1F}

$$\ddot{\phi}_{1F} = \ddot{\phi}_{2P} X_4 X_5 + \dot{\phi}^2_{2P} X_4 X_6 \qquad (106)$$

where

$$X_4 = \frac{1}{\left(D_{1F} \cos \phi_1 - E_{1F} \sin \phi_1\right)}$$

$$X_5 = F_{1FD} - D_{1FD} \sin \phi_1 - E_{1FD} \cos \phi_1$$

$$X_6 = F_{1FDD} - D_{1FDD} \sin \phi_1 - E_{1FDD} \cos \phi_1$$

$$+ \text{DER1F} \left[- 2 D_{1FD} \cos \phi_1 + 2 E_{1FD} \sin \phi_1\right]$$

$$+ \text{DER2F}^2 \left[D_{1F} \sin \phi_1 + E_{1F} \cos \phi_1\right]$$

INVERSE KINEMATICS OF MESH NO. 2
(Angle ϕ is Input and Angle ϕ_{2G} is Output)

Round-On-Round Phase of Motion

Determination of Angle ϕ_{2G} in Terms of Angle ϕ

The component loop equations 53 and 54 are again used to substitute into equation 55; i.e.,

$$\sin^2 \lambda + \cos^2 \lambda = 1$$

This leads to

$$L_2^2 - a_{P2}^2 - b_2^2 - a_{G2}^2 - 2\, a_{P2}\, b_2 \cos\left(\phi + \delta_{P2} - \beta_2\right)$$

$$= \sin \phi_{2G} \left[2\, a_{P2}\, a_{G2} \left\{\cos\left(\phi + \delta_{P2}\right) \sin \delta_{G2} - \sin\left(\phi + \delta_{P2}\right) \cos \delta_{G2}\right\}\right.$$

$$\left. + 2\, b_2\, a_{G2} \left\{\cos \beta_2 \sin \delta_{G2} - \sin \beta_2 \cos \delta_{G2}\right\}\right]$$

$$+ \cos \phi_{2G} \left[- 2\, a_{P2}\, a_{G2} \left\{\cos\left(\phi + \delta_{P2}\right) \cos \delta_{G2} + \sin\left(\phi + \delta_{P2}\right) \sin \delta_{G2}\right.\right.$$

$$\left.\left. - 2\, b_2\, a_{G2} \left\{\cos \beta_2 \cos \delta_{G2} + \sin \beta_2 \sin \delta_{G2}\right\}\right] \tag{108}$$

Further simplification furnishes

$$L_2^2 - a_{P2}^2 - b_2^2 - a_{G2}^2 - 2\, a_{P2}\, b_2 \cos\left(\phi + \delta_{P2} - \beta_2\right)$$

$$= \sin \phi_{2G} \left[- 2\, a_{P2}\, a_{G2} \sin\left(\phi + \delta_{P2} - \delta_{G2}\right) - 2\, a_{G2}\, b_2 \sin\left(\beta_2 - \delta_{G2}\right)\right]$$

$$- \cos \phi_{2G} \left[- 2\, a_{P2}\, a_{G2} \cos\left(\phi + \delta_{P2} - \delta_{G2}\right) - 2\, a_{G2}\, b_2 \cos\left(\beta_2 - \delta_{G2}\right)\right] \tag{109}$$

The above then gives

$$D_{2R} \sin \phi_{2G} + E_{2R} \cos \phi_{2G} = F_{2R} \tag{110}$$

where

$$D_{2R} = -2\, a_{P2}\, a_{G2} \sin\left(\phi + \delta_{P2} - \delta_{G2}\right) - 2\, a_{G2}\, b_2 \sin\left(\beta_2 - \delta_{G2}\right)$$

$$E_{2R} = -2\, a_{P2}\, a_{G2} \cos\left(\phi + \delta_{P2} - \delta_{G2}\right) - 2\, a_{G2}\, b_2 \cos\left(\beta_2 - \delta_{G2}\right)$$

$$F_{2R} = L_2^2 - a_{G2}^2 - a_{P2}^2 - b_2^2 - 2\, a_{P2}\, b_2 \cos\left(\phi + \delta_{P2} - \beta_2\right)$$

Equation 110 is solved for the angle ϕ_{2G} in the manner shown in reference 1; i.e.,

$$\phi_{2G} = 2 \tan^{-1} \frac{D_{2R} \pm \sqrt{D_{2R}^2 + E_{2R}^2 - F_{2R}^2}}{E_{2R} + F_{2R}} \tag{111}$$

Determination of Angular Velocity $\dot{\phi}_{2GR}$ in Terms of Angular Velocity and Angles ϕ_{2G} and ϕ

Implicit differentiation of equation 110 gives

$$D_{2R}\, \dot{\phi}_{2G} \cos \phi_{2G} + D_{2RD}\, \dot{\phi} \sin \phi_{2G} - E_{2R}\, \dot{\phi}_{2G} \sin \phi_{2G}$$

$$- E_{2RD}\, \dot{\phi} \cos \phi_{2G} = F_{2RD}\, \dot{\phi} \tag{112}$$

where

$$D_{2RD} = -2\, a_{P2}\, a_{G2} \cos\left(\phi + \delta_{P2} - \delta_{G2}\right)$$

$$E_{2RD} = 2\, a_{P2}\, a_{G2} \sin\left(\phi + \delta_{P2} - \delta_{G2}\right)$$

$$F_{2RD} = 2\, a_{P2}\, b_2 \sin\left(\phi + \delta_{P2} - \beta_2\right)$$

Then

$$\dot{\phi}_{2G} \left[D_{2R} \cos \phi_{2G} - E_{2R} \sin \phi_{2G}\right]$$

$$= \dot{\phi} \left[F_{2RD} - D_{2RD} \sin \phi_{2G} - E_{2RD} \cos \phi_{2G}\right] \tag{113}$$

After introducing the additional subscript R to indicate the round-on-round phase, the angular velocity $\dot{\phi}_{2GR}$ is given by

$$\dot{\phi}_{2GR} = \dot{\phi}\ \mathrm{DER2R} \tag{114}$$

where

$$DER2R = \frac{F_{2RD} - D_{2RD} \sin \phi_{2G} - E_{2RD} \cos \phi_{2G}}{D_{2R} \cos \phi_{2G} - E_{2R} \sin \phi_{2G}}$$

Determination of Angular Acceleration $\ddot{\phi}_{2GR}$ in Terms of Angular Acceleration $\ddot{\phi}$, Angular Velocity $\dot{\phi}$, and Angles ϕ and ϕ_{2G}

Further differentiation of equation 113 furnishes

$$\begin{aligned}
&\ddot{\phi}_{2G} \left[D_{2R} \cos \phi_{2G} - E_{2R} \sin \phi_{2G}\right] \\
&+ \dot{\phi}_{2G} \dot{\phi} \left[D_{2RD} \cos \phi_{2G} - E_{2RD} \sin \phi_{2G}\right] \\
&+ \dot{\phi}_{2G}^2 \left[- D_{2R} \sin \phi_{2G} - E_{2R} \cos \phi_{2G}\right] \\
&= \ddot{\phi} \left[F_{2RD} - D_{2RD} \sin \phi_{2G} - E_{2RD} \cos \phi_{2G}\right] \\
&+ \dot{\phi}^2 \left[F_{2RDD} - D_{2RDD} \sin \phi_{2G} - E_{2RDD} \cos \phi_{2G}\right] \\
&+ \dot{\phi}\dot{\phi}_{2G} \left[- D_{2RD} \cos \phi_{2G} + E_{2RD} \sin \phi_{2G}\right] \qquad (115)
\end{aligned}$$

where

$$D_{2RDD} = 2\, a_{P2}\, a_{G2} \sin \left(\phi + \delta_{P2} - \delta_{G2}\right)$$

$$E_{2RDD} = 2\, a_{P2}\, a_{G2} \cos \left(\phi + \delta_{P2} - \delta_{G2}\right)$$

$$F_{2RDD} = 2\, a_{P2}\, b_2 \cos \left(\phi + \delta_{P2} - \beta_2\right)$$

Substitution of equation 114 into equation 115 and the introduction of the additional subscript R, for round-on-round motion, results in the following expression for angular acceleration ϕ_{2GR}:

$$\ddot{\phi}_{2GR} = \ddot{\phi}\, X_7 X_8 + \dot{\phi}^2\, X_7 X_9 \qquad (116)$$

where

$$X_7 = \frac{1}{D_{2R} \cos \phi_{2G} - E_{2R} \sin \phi_{2G}}$$

$$X_8 = F_{2RD} - D_{2RD} \sin \phi_{2G} - E_{2RD} \cos \phi_{2G}$$

$$X_9 = F_{2RDD} - D_{2RDD} \sin \phi_{2G} - E_{2RDD} \cos \phi_{2G}$$
$$+ DER2R \left[- 2 D_{2RD} \cos \phi_{2G} + 2 E_{2RD} \sin \phi_{2G}\right]$$
$$+ DER2R^2 \left[D_{2R} \sin \phi_{2G} + E_{2R} \cos \phi_{2G}\right]$$

Round-On-Flat Phase of Motion

Determination of Angle ϕ_{2G} in Terms of Angle ϕ

When equation 68 for g_2 is substituted into equation 66, the result is

$$\left[a_{G2} \cos (\phi_{2G} + \delta_{G2}) - \rho_{G2} \sin (\phi - \alpha_{P2}) - b_2 \cos \beta_2\right] \sin (\phi - \alpha_{P2})$$
$$- \left[a_{G2} \sin (\phi_{2G} + \delta_{G2}) + \rho_{G2} \cos (\phi - \alpha_{P2}) - b_2 \sin \beta_2\right] \cos (\phi - \alpha_{P2}) = 0 \tag{117}$$

Now expand the terms containing the angle ϕ_{2G} trigonometrically

$$\left[a_{G2} (\cos \phi_{2G} \cos \delta_{G2} - \sin \phi_{2G} \sin \delta_{G2}) - \rho_{G2} \sin (\phi - \alpha_{P2})\right.$$
$$\left.- b_2 \cos \beta_2\right] \sin (\phi - \alpha_{P2}) - \left[a_{G2} (\sin \phi_{2G} \cos \delta_{G2} + \cos \phi_{2G} \sin \delta_{G2})\right.$$
$$\left.+ \rho_{G2} \cos (\phi - \alpha_{P2}) - b_2 \sin \beta_2\right] \cos (\phi - \alpha_{P2}) = 0 \tag{118}$$

Further rearrangement furnishes

$$\left[- a_{G2} \sin (\delta_{G2} - \phi + \alpha_{P2})\right] \cos \phi_{2G} + \left[- a_{G2} \cos (\delta_{G2} - \phi + \alpha_{P2})\right] \sin \phi_2$$
$$- \rho_{G2} + b_2 \sin (\beta_2 - \phi + \alpha_{P2}) = 0 \tag{119}$$

or

$$D_{2F} \sin \phi_{2G} + E_{2F} \cos \phi_{2G} = F_{2F} \tag{120}$$

where

$$D_{2F} = -a_{G2} \cos(\phi - \alpha_{P2} - \delta_{G2})$$

$$E_{2F} = a_{G2} \sin(\phi - \alpha_{P2} - \delta_{G2})$$

$$F_{2F} = \rho_{G2} + b_2 \sin(\phi - \alpha_{P2} - \beta_2)$$

Solution of equation 120 for the angle ϕ_{2G}, according to the method described in reference 1, leads to

$$\phi_{2G} = 2 \tan^{-1} \frac{D_{2F} \pm \sqrt{D_{2F}^2 + E_{2F}^2 - F_{2F}^2}}{E_{2F} + F_{2F}} \tag{121}$$

Determination of Angular Velocity $\dot{\phi}_{2GF}$ in Terms of Angular Velocity $\dot{\phi}$ and Angles ϕ_{2G} and ϕ

Implicit differentiation of equation 120 with respect to time gives

$$D_{2F}\, \dot{\phi}_{2G} \cos \phi_{2G} + D_{2FD}\, \dot{\phi} \sin \phi_{2G} - E_{2F}\, \dot{\phi}_{2G} \sin \phi_{2G}$$

$$+ E_{2FD}\, \dot{\phi} \cos \phi_{2G} = F_{2FD}\, \dot{\phi} \tag{122}$$

where

$$D_{2FD} = a_{G2} \sin(\phi - \alpha_{P2} - \delta_{G2})$$

$$E_{2FD} = a_{G2} \cos(\phi - \alpha_{P2} - \delta_{G2})$$

$$F_{2FD} = b_2 \cos(\phi - \alpha_{P2} - \beta_2)$$

Then

$$\dot{\phi}_{2G} (D_{2F} \cos \phi_{2G} - E_{2F} \sin \phi_{2G}) = \dot{\phi} (F_{2FD} - D_{2FD} \sin \phi_{2G} - E_{2FD} \cos \phi_{2G}) \tag{123}$$

Finally, after introduction of the additional subscript F, the angular velocity $\dot{\phi}_{2GF}$, for the round-on-flat phase of motion of mesh no. 1 is expressed by

$$\dot{\phi}_{2GF} = \dot{\phi}\ \mathrm{DER2F} \tag{124}$$

where

$$\mathrm{DER2F} = \frac{F_{2FD} - D_{2FD}\sin\phi_{2G} - E_{2FD}\cos\phi_{2G}}{D_{2F}\cos\phi_{2G} - E_{2F}\sin\phi_{2G}}$$

Determination of Angular Acceleration $\ddot{\phi}_{2GR}$ in Terms of Angular Acceleration $\ddot{\phi}_1$, Angular Velocity $\dot{\phi}_1$, and Angles ϕ and ϕ_{2G}

Implicit differentiation of equation 123 with respect to time furnishes

$$\begin{aligned}
&\ddot{\phi}_{2G}\left[D_{2F}\cos\phi_{2G} - E_{2F}\sin\phi_{2G}\right] \\
&+ \dot{\phi}_{2G}\,\dot{\phi}\left[D_{2FD}\cos\phi_{2G} - E_{2FD}\sin\phi_{2G}\right] \\
&+ \dot{\phi}_{2G}^{2}\left[-D_{2F}\sin\phi_{2G} - E_{2F}\cos\phi_{2G}\right] \\
&= \ddot{\phi}\left[F_{2FD} - D_{2FD}\sin\phi_{2G} - E_{2FD}\cos\phi_{2G}\right] \\
&+ \dot{\phi}^{2}\left[F_{2FDD} - D_{2FDD}\sin\phi_{2G} - E_{2FDD}\cos\phi_{2G}\right] \\
&+ \dot{\phi}\,\dot{\phi}_{2G}\left[-D_{2FD}\cos\phi_{2G} + E_{2FD}\sin\phi_{2G}\right]
\end{aligned} \tag{125}$$

where

$$D_{2FDD} = a_{G2}\cos\left(\phi - \alpha_{P2} - \delta_{G2}\right)$$

$$E_{2FDD} = -a_{G2}\sin\left(\phi - \alpha_{P2} - \delta_{G2}\right)$$

$$F_{2FDD} = -a_{G2}\sin\left(\phi - \alpha_{P2} - \beta_2\right)$$

Substitution of equation 124, and further use of the additional subscript F for round-on-flat motion, leads to the following expression for the angular acceleration $\ddot{\phi}_{2GF}$:

$$\ddot{\phi}_{2GF} = \ddot{\phi}\, X_{10}X_{11} + \dot{\phi}^2\, X_{10}X_{12} \tag{126}$$

where

$$X_{10} = \frac{1}{D_{2F}\cos\phi_{2G} - E_{2F}\sin\phi_{2G}}$$

and

$$X_{11} = F_{2FD} - D_{2FD}\sin\phi_{2G} - E_{2FD}\cos\phi_{2G}$$

$$\begin{aligned} X_{12} = {} & F_{2FDD} - D_{2FDD}\sin\phi_{2G} - E_{2FDD}\cos\phi_{2G} \\ & + \mathrm{DER2F}\left[-2F_{2FD}\cos\phi_{2G} + E_{2FD}\sin\phi_{2G}\right] \\ & + \mathrm{DER2F}^2\left[D_{2F}\sin\phi_{2G} + E_{2F}\cos\phi_{2G}\right] \end{aligned}$$

REFERENCE

1. G. G. Lowen and F. R. Tepper, "Fuze Gear Train Analysis," Technical Report ARLCD-TR-79030, ARRADCOM, Dover, NJ, December 1979.

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